Quadrilaterals I
Geometry Research Honors

Parallelograms

8.3.1 $TYUI$ is a parallelogram with $TY = 6$ and $YU = 8$.

(a) What is the perimeter of $TYUI$?

(b) Do we have enough information to find the area of $TYUI$?

8.3.2 $WXYZ$ is a parallelogram. Prove that $\angle W = \angle Y$.

8.3.3 In quadrilateral $WORK$, $WO = RK$ and $\overline{WO} \parallel \overline{RK}$. Prove that $WORK$ is a parallelogram.

8.3.4 Use a clever dissection of a parallelogram to turn it into a rectangle and prove that the area of the parallelogram is its base times its height. **Hints:** 394

8.3.5 Is it possible for a parallelogram $ENTU$ to have $EN = 4$, $EU = 12$, and for sides $\overline{EU}$ and $\overline{NT}$ to be 5 units apart? Why or why not? **Hints:** 528

8.3.6 In this Exercise, we give another way of deriving the length of the median and area of a trapezoid.

Let $ABCD$ be a trapezoid with bases $\overline{AB}$ and $\overline{CD}$.

(a) Extend $\overline{AB}$ past B to A' such that $BA' = CD$, and extend $\overline{DC}$ past C to D' such that $CD' = AB$. Prove that $AA'D'D$ is a parallelogram.

(b) Find the area of $[AA'D'D]$ in terms of AB, CD, and h, the distance between $\overline{AB}$ and $\overline{CD}$. Find the relationship between $[AA'D'D]$ and $[ABCD]$, and then find $[ABCD]$ itself. **Hints:** 341

(c) Let M, N, and M' be the midpoints of $\overline{AD}$, $\overline{BC}$, and $\overline{A'D'}$, respectively. Prove that $MM' = AB + CD$, and prove that $MN = MM'/2$. Conclude that $MN = (AB + CD)/2$.

8.3.7★ The diagonals of convex quadrilateral $ABCD$ meet at E. Prove that the centers of the circumcircles of $\triangle ABE$, $\triangle BCE$, $\triangle CDE$, and $\triangle DAE$ are the vertices of a parallelogram. **Hints:** 80

Rhombi

8.4.1 $PQRS$ is a rhombus with diagonals $PR = 6$ and $QS = 12$. Find the area and the perimeter of $PQRS$.

8.4.2 $WXYZ$ is a rhombus with $WX = 50$ and $WY = 96$.

(a) Find XZ.

(b) Find $[WXYZ]$.

(c) Find the distance between $\overline{WX}$ and $\overline{YZ}$. **Hints:** 586

8.4.3 Diagonals $\overline{AC}$ and $\overline{BD}$ of quadrilateral $ABCD$ are perpendicular. Prove that $[ABCD] = (AC)(BD)/2$. **Hints:** 438

8.4.4 $TUVW$ is a rhombus with $TU = 10$ and $\angle TUV = 60°$.

(a) Show that $\angle TUW = 30°$.

(b) Find $[TUVW]$.

Rectangles

8.5.1 $POST$ is a rectangle with $PO = 8$ and $OS = 12$.

(a) Find the perimeter of $POST$.

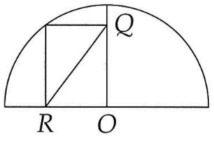
(b) Find PS.

(c) Find $[POST]$.

8.5.2 The length of a rectangle is one less than twice its width. If the perimeter of the rectangle is 36, what is the area of the rectangle?

8.5.3 Diagonals $\overline{WR}$ and $\overline{ET}$ of rectangle $WERT$ meet at Y. Given that $\angle WYE = x$, find $\angle ERY$ and $\angle YRT$ in terms of x.

8.5.4 A semicircle with center O has a radius of 9 cm. What is the number of centimeters in the length of $\overline{RQ}$, a diagonal of the rectangle shown? *(Source: MATHCOUNTS)*



8.5.5 I have a 36 inch by 24 inch rectangular painting. I would like to place a frame that is 2 inches wide around the painting. If the material for the frame costs \$1.50 per square inch, how much will the frame cost?

8.5.6 What kind of quadrilateral do we get when we connect the midpoints of the sides of a rectangle? (Prove your answer!)

8.5.7★ $EFGH$ is a rectangle with area 48. If $EGJI$ is a rectangle such that H is on $\overline{JI}$, what is the area of $EGJI$? **Hints:** 434

Squares

8.6.1 The area of square $EFGH$ is 80. Find EF and EG.

8.6.2 M is the midpoint of $\overline{AB}$ on square $ABCD$. If $\overline{AC}$ and $\overline{BD}$ meet at O, and $MO = 4$, what is the area of square $ABCD$?

8.6.3 The diagonals of square $TYUI$ meet at M. Point K is on side $\overline{TY}$ such that $TK = TM$. Find $\angle MTK$ and $\angle TMK$.

8.6.4 Show that a rectangle with perpendicular diagonals must be a square.

8.6.5 $ABCD$ and $ACFG$ are squares. Find $[ACFG]/[ABCD]$. **Hints:** 236

8.6.6 Point E is inside square $ABCD$ such that $\triangle ABE$ is equilateral. Given that $AB = 4$, find the following:

(a) AE.

(b) $[ABCD]$.

(c) $[ABE]$.

(d) $\angle DAE$ and $\angle DEA$.

(e) The area inside $ABCD$ but outside $\triangle ABE$.

(f) CE.